N90-10095

EFFICIENCY AND CAPABILITIES OF MULTI-BODY SIMULATIONS

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ABSTRACT

Simulation efficiency and capability go hand in hand. The more capability you have the lower the efficiency will be. ~~Section 1 of this paper discusses~~ The efficiency and ~~section 2 deals with~~ capabilities. The lesson ~~we have~~ learned about generic simulation is: Don't rule out any capabilities at the beginning but keep each one on a switch so it can be bypassed when warranted by a specific application.

1. EFFICIENCY

Efficiency means different things to different people. For the person running simulations interactively on a terminal quick turn around time is efficiency. For the person making 10,000 Monte-Carlo runs low cost is efficiency. For the person running real time simulations minimum CPU time is efficiency.

Three aspects of a simulation should be considered when dealing with efficiency; hardware, software and modeling.

Hardware A fast processor will reduce CPU time for a given simulation but this doesn't necessarily equate to improved efficiency. For example, the Monte-Carlo simulation may take 10 minutes on a super computer and 2 weeks on a PC but if time is free on the PC then that may be an efficient solution. We will not discuss hardware related issues except for two points. 1.) Fast hardware is of primary importance to the real time simulation because it means higher fidelity models can be incorporated 2.) Vector processors and parallel processors should use custom algorithms that take full advantage of the special machine architecture.

Software A fast algorithm will also reduce CPU time but again this doesn't necessarily equate to improved efficiency. For example, it is generally accepted that an ad-hoc simulation is much faster than a generic simulation. The cost of developing and testing the ad-hoc simulation may exceed the run time saving thereby reducing overall efficiency.

Recent work in the area of symbolic programming has shown that significant savings can be achieved by symbolicaly forming the equation of motion and numerically solving them. Other algorithms have been proposed that promise similar savings. There is one point that software developers should keep in mind. With generic simulations the user must have complete flexibility in retaining or deleting different parts of his model. This is because generic simulations are often used for model development and validation. In that environment an analyst will add or delete certain features to determine the effect on performance and whether or not the feature should be retained in the model.

More on this subject in section 2.

Modeling This is the domain of the simulation user and the area in which many improvements in efficiency can be made. For example, deleting a high order mode in a flexible body model has a compound effect. It reduces the model complexity and at the same time allows a bigger integration step size both of which reduce run time. Often times the reduced fidelity is justified by the savings in run time.

The point to be made is that the analyst is the end authority on the "correct" model for a given application. The more flexibility he has in changing his model the easier it is for him to select the best model for the job.

2. CAPABILITIES

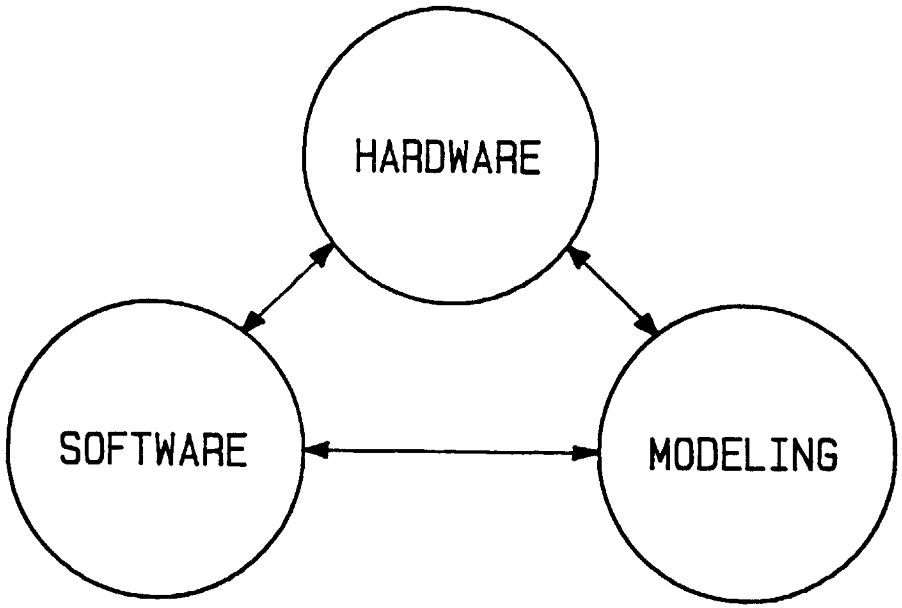
Capability in our context is synonymous with flexibility and not with complexity. A simulation may be very detailed and complex but if it can't be changed then it's only useful in a narrow range of applications and has limited capability.

In our experience with TREETOPS and DCAP we have found that it is much easier to generate a model and obtain a response than it is to predict the correct response. In other words, when we don't get the expected response the simulation is usually correct and our expectation is wrong. This is not entirely unexpected because it is very difficult, even for an expert, to solve the equations of anything but the simplest dynamical systems. The solution to this dilemma is flexibility. Start with simple models that have known analytic solutions. Then add complexity one step at a time while gaining confidence in your model and insight into the behavior of your system.

For multibody systems with flexible bodies the same arguments apply but the complexity of the model increases more rapidly than for rigid bodies. The person doing software development makes assumptions that simplify the resulting equations of motion. If this is done carelessly then terms are dropped that may prove essential in specific applications. On the other hand, if simplifications are not made then the computation burden becomes too great.

The lesson we learned is that you must retain as many terms as possible in the kinematics but they must have associated switches so you can easily add or delete them from a specific application. This is done for two reasons. 1.) to give you insight into the effect of various model elements on system response and 2.) to allow the selection of the most efficient model for a given application.

SIMULATION EFFICIENCY



- BYPASS TERMS
- MULTI-RATE ALGORITHMS
- SYMBOLIC PROGRAMMING
- ADHOC SIMULATION
-

- BYPASS TERMS
- INTEGRATION TYPE & STEP SIZE
- REDUCED ORDER
-
-

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SPEED-UP OPTIONS

	COORDINATE TRANSFORMATIONS	MASS MATRIX FORMULATION (M)	NON-LINEAR TERMS $(\underline{\omega} \times \underline{\underline{I}} \cdot \underline{\omega})$	CONSTRAINT FORMULATION (A)
PERFORM ALL COMPUTATIONS				
COMPUTE ONLY ON FIRST PASS OF R-K INTEGRATION				
COMPUTE ONLY ON FIRST PASS OF NTH R-K STEP				
BYPASS COMPUTATIONS				

TREETOPS SOFTWARE IMPROVEMENTS

	EQUATION FORMULATION	EQUATION SOLUTION	PROCESSING HARDWARE
CURRENT STATUS	NUMERIC	NUMERIC	SERIAL
FIRST STEP	SYMBOLIC	NUMERIC	SERIAL
SECOND STEP	SYMBOLIC	NUMERIC	PARALLEL

$$M\ddot{q} = f + A^T \lambda$$

$$A\dot{q} = B$$

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SIMULATION CAPABILITY-MENUS

BODIES	SENSORS	ACTUATORS	CONSTRAINTS	DEVICES	CONTROLLERS
1. RIGID	1. RATE GYRO	1. REACTION JET	1. CLOSED LOOP	1. SPRINGS	1. CONTINUOUS
2. FLEXIBLE	2. RESOLVER	2. HYDRAULIC CYLINDER	2. VELOCITY-TIME	2. DAMPERS	2. DISCRETE
	3. ANGULAR ACCELEROMETER	3. REACTION WHEEL	3. VELOCITY -DIRECTION	3. COULOMB DAMPER	3. BLOCK DIAGRAM (FREQUENCY DOMAIN)
	4. VELOCITY	4. TORQUE MOTOR	4. RATE-TIME	4. QUADRATIC SPRING/DAMPER	4. MATRIX (STATE SPACE)
	5. POSITION	5. MOMENT	5. RATE-DIRECTION	5. SOLID DAMPER	5. USER
	6. ACCELEROMETER	6. BRAKE	6. CUT JPINT	6. HARDSTOP	
	7. TACHOMETER	7. LOCK		7. CONTACT SPRINGS	
	8. INTEGRATING RATE GYRO	8. SINGLE GIMBAL CMG			
	9. SUN SENSOR	9. DOUBLE GIMBAL CMG			
	10. STAR SENSOR	10. MAGNETIC			
	11. IMU				
	12. POSITION VECTOR				
	13. VELOCITY VECTOR				

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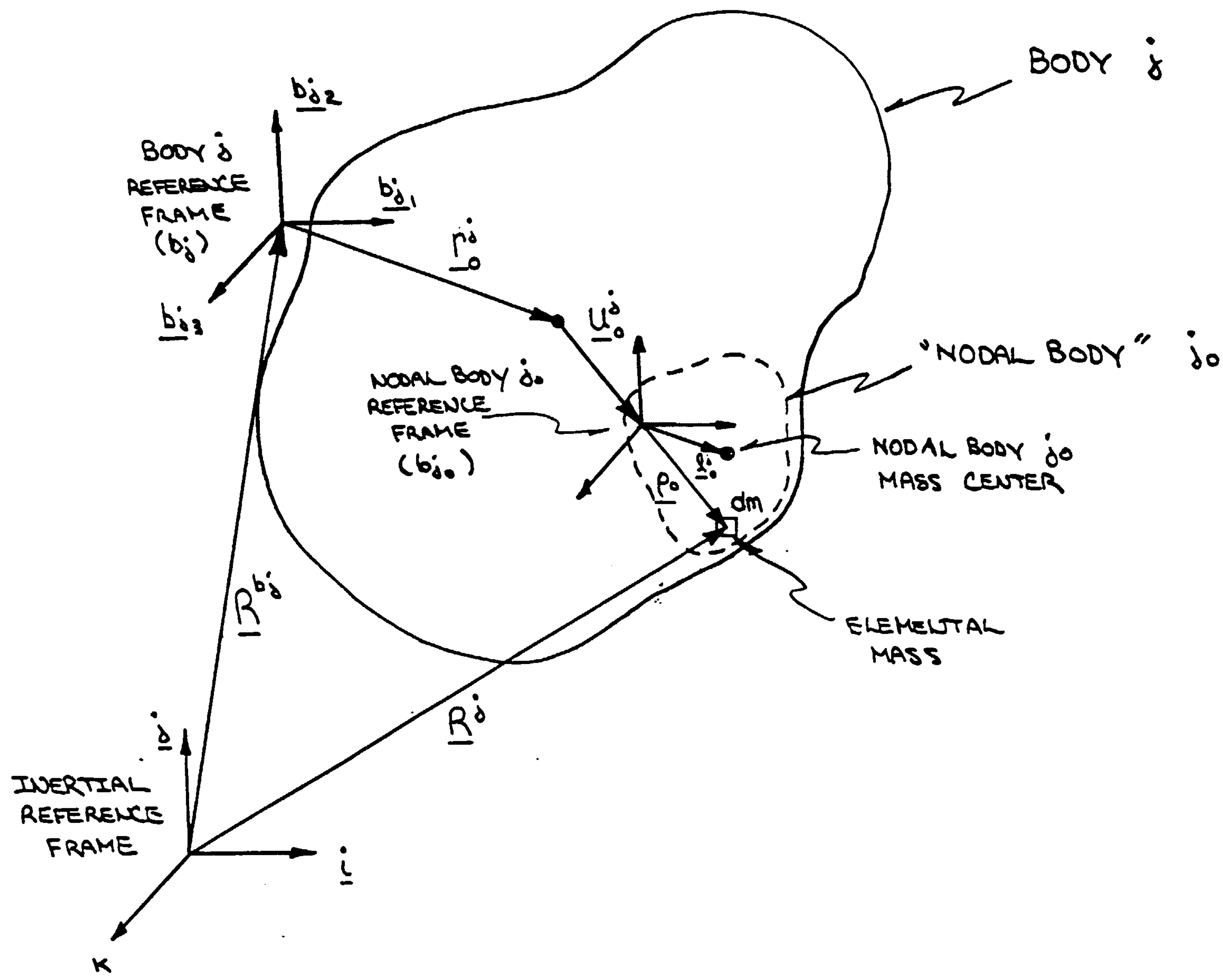
SWITCHES FOR MODAL DATA

- HIGH LEVEL
  - LUMPED MASS SWITCH
- MID LEVEL
  - FIRST ORDER SWITCH
  - SECOND ORDER SWITCH
  - THIRD ORDER SWITCH
- LOW LEVEL
  - ONE SWITCH FOR EACH TERM

FLEXIBLE BODY DISPLACEMENT FIELD



EQUATIONS OF MOTION

$$f_K + f_K^{*} = 0 \quad ; \quad K = 1, \dots, N$$

$$f_K = \sum_{j=1}^{NB} \left\{ \int_j \underline{V}_K^{j} \cdot d\underline{f}^{j} \right\}$$

$$-f_K^{*} = \sum_{j=1}^{NB} \left\{ \sum_{o=1}^{NNB_j} \int_{j_o} \underline{V}_K^{j} \cdot \underline{\ddot{R}}^{j} dm \right\}$$

FROM NEWTON'S LAW

$$\int_{b_j} d\underline{f}^{j} - \underline{\ddot{R}}^{j} dm = \underline{0}$$

NOTE: OPEN DOT DENOTES LOCAL TIME DERIVATIVE, SOLID DOT DENOTES INERTIAL TIME DERIVATIVE.

$$\underline{V}_K^{j} = \underline{V}_K^{b_j} + \underline{\omega}_K^{j} \times (\underline{r}_o^{j} + \underline{u}_o^{j} + \underline{\rho}_o) + \underline{V}_K^{\eta_j}$$

$$\underline{\ddot{R}}^{j} = \underline{\ddot{R}}^{b_j} + \underline{\dot{\omega}}^{j} \times (\underline{r}_o^{j} + \underline{u}_o^{j} + \underline{\rho}_o) + \underline{\mathring{\mathring{u}}}_o^{j} + \underline{\mathring{\mathring{u}}}_o^{\prime j} \times \underline{\rho}_o + 2\,\underline{\omega}^{j} \times \underline{\mathring{u}}_o^{j}$$

$$+ (\underline{\omega}^{j} \times \underline{\mathring{u}}_o^{\prime j}) \times \underline{\rho}_o + \underline{\omega}^{j} \times (\underline{\omega}^{j} \times (\underline{r}_o^{j} + \underline{u}_o^{j} + \underline{\rho}_o))$$

$$+ \underline{\omega}^{j} \times (\underline{\mathring{u}}_o^{\prime j} \times \underline{\rho}_o) + \underline{\mathring{u}}_o^{\prime j} \times (\underline{\omega}^{j} \times \underline{\rho}_o) + \underline{\mathring{u}}_o^{\prime j} \times (\underline{\mathring{u}}_o^{\prime j} \times \underline{\rho}_o)$$

COEFFICIENTS OF GENERALIZED SPEEDS

TRANSLATIONAL D.O.F.

LET K CORRESPOND TO THE i-TH TRANSLATIONAL D.O.F. OF THE g-TH HINGE.

$$\underline{V}_K^{b_j} = \begin{cases} \underline{g}_i^g \,;\; j \in E(g) \\ \underline{0} \,;\; \text{OTHERWISE} \end{cases}$$

$$\underline{\omega}_K^j = \underline{0}$$

$$\underline{V}_K^{\dot{q}j} = \underline{0}$$

ROTATIONAL D.O.F.

LET K CORRESPOND TO THE i-TH ROTATIONAL D.O.F. OF THE g-TH HINGE.

$$\underline{V}_K^{b_j} = \begin{cases} \underline{\ell}_i^g(\theta^g) \times \left[{}^g\underline{\chi}^j - (\underline{r}_{hg} + \underline{u}_{hg})\right] \,;\; j \in E(g) \\ \underline{0} \,;\; \text{OTHERWISE} \end{cases}$$

$$\underline{\omega}_K^j = \begin{cases} \underline{\ell}_i^g(\theta^g) \,;\; j \in E(g) \\ \underline{0} \,;\; \text{OTHERWISE} \end{cases}$$

$$\underline{V}_K^{\dot{q}j} = \underline{0}$$

MODAL D.O.F.

LET K CORRESPOND TO THE i-TH MODAL D.O.F. OF THE g-TH BODY.

$$\underline{V}_K^{b_j} = \begin{cases} -\underline{\phi}_i'^g(\underline{r}_{hg}) \times {}^{hg}\underline{\chi}^{b_j} + \underline{\phi}_i'^g(\underline{r}_{p(g)}) \times {}^{p(g)}\underline{S}^{b_j}(1-\delta_{jg}) \\ \quad + \underline{\phi}_i^g(\underline{r}_{Kg})(1-\delta_{jg}) - \underline{\phi}_i^g(\underline{r}_{hg}) \,;\; j \in E(g) \\ \underline{0} \,;\; \text{OTHERWISE} \end{cases}$$

$$\underline{\omega}_K^j = \begin{cases} -\underline{\phi}_i'^g(\underline{r}_{hg}) + \underline{\phi}_i'^g(\underline{r}_{Kg})(1-\delta_{jg}) \,;\; j \in E(g) \\ 0 \,;\; \text{OTHERWISE} \end{cases}$$

$$\underline{V}_K^{\dot{q}j} = \begin{cases} \underline{\phi}_i^g(\underline{r}_o^g) + \underline{\phi}_i'^g(\underline{r}_o^g) \times \underline{\rho}_o \,;\; j = g \\ \underline{0} \,;\; \text{OTHERWISE} \end{cases}$$

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$\underline{g}_i^g \equiv$ i-TH TRANSLATION AXIS OF THE g-TH HINGE, FIXED IN $L(g)$, BODY INBOARD OF THE g-TH BODY

$\underline{l}_i^g(\theta^g) \equiv$ i-TH ROTATION AXIS (EULER AXIS) OF THE g-TH HINGE

${}^g\underline{\chi}^j \equiv$ VECTOR LOCATING BODY j REFERENCE WRT BODY g REFERENCE

$\underline{r}_{hg} \equiv$ VECTOR LOCATING UNDEFORMED HINGE ATTACH POINT ON BODY g WRT BODY g REFERENCE.

$\underline{U}_{hg} \equiv$ DEFORMATION AT $\underline{r}_{hg}$ ON BODY g $\left(\underline{U}_{hg} = \sum_{i=1}^{NM_g} \underline{\phi}_i^g(\underline{r}_{hg})\eta_i^g \right)$

${}^{hg}\underline{z}^{bj} \equiv$ VECTOR LOCATING BODY j REFERENCE WRT BODY g HINGE ATTACH POINT ON BODY g.

${}^{P(g)}\underline{S}^{bj} \equiv$ VECTOR FROM POINT $P(g)$ ON BODY g (LEADING TO BODY j) TO BODY j REFERENCE. (IF $j = g$, ${}^{P(g)}\underline{S}^{bj} = \underline{0}$)

$E(g) \equiv$ SET OF ALL BODIES OUTBOARD OF THE g-TH BODY INCLUDING BODY g

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"GENERALIZED INERTIAL FORCE"
(EXPRESSION FOR $-f^*_k$)

$$-f^*_k = \sum_{j=1}^{NB} \left\{ m_j \left(\ddot{\underline{R}}^{b_j} + \ddot{\underline{L}}_j \right) \cdot \underline{V}^{b_j}_k + \left(\dot{\underline{H}}^j + m_j \underline{L}_j \times \ddot{\underline{R}}^{b_j} \right) \cdot \underline{\omega}^j_k + \sum_{o=1}^{NNB_j} \int_{b_{jo}} \underline{V}^{q_j}_k \cdot \ddot{\underline{R}}_j \, dm \right\}$$

MODAL TERMS

K CORRESPONDING TO THE i-TH MODAL D.O.F. OF THE j-TH BODY

(I.) $m_j \underline{\alpha}^j_i \cdot \ddot{\underline{R}}^{b_j}$

(II.) $\left(\underline{h}^j_i + \sum_{k=1}^{NM_j} \underline{Y}^j_{ki} \eta^j_k + \sum_{k=1}^{NM_j} \sum_{m=1}^{NM_j} \underline{Z}^j_{mki} \eta^j_m \eta^j_k \right) \cdot \dot{\underline{\omega}}^j$

(III.) $2 \underline{\omega}^j \cdot \left(\sum_{k=1}^{NM_j} \underline{Y}^j_{ki} \dot{\eta}^j_k + \sum_{k=1}^{NM_j} \sum_{m=1}^{NM_j} \underline{Y}^j_{mki} \eta^j_m \dot{\eta}^j_k \right)$

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(IV.) MODAL MASS (ASSUMED BODY BASIS)

— SCALAR REPRESENTATION FOR "LUMPED APPROACH"

$$\left\{ \sum_{o=1}^{NNB^j} \left[\{\phi^j_{o_i}\}^T \{\phi'^j_{o_i}\}^T \right] \left[\begin{array}{c|c} \left[m^j_{o\,\mathrm{DIAGONAL}} \right] & \left[-m^j_o \tilde{l}^j_o \right] \\ \hline \left[m^j_o \tilde{l}^j_o \right] & \left[J^{b_j o} + \tilde{u}'^j_o J^{b_j o} - J^{b_j o} \tilde{u}'^j_o - \tilde{u}'^j_o J^{b_j o} \tilde{u}'^j_o \right] \end{array} \right] \left[\begin{array}{c} \{\phi^j_{o_k}\} \\ \hline \{\phi'^j_{o_k}\} \end{array} \right] \right\} \ddot{\eta}^j_k$$

(V.) $\underline{\omega}^j \cdot \left[\underline{\underline{W}}^j_i + \sum_{K=1}^{NM^j} \underline{\underline{W}}^j_{Ki} \eta^j_K + \sum_{K=1}^{NM^j} \sum_{M=1}^{NM^j} \underline{\underline{W}}^j_{MKi} \eta^j_m \eta^j_K \right] \cdot \underline{\omega}^j$

$$\sum_{o=1}^{NNB^j} (\underline{\omega}^j + \underline{\mathring{u}}'^j_o) \cdot \left(\underline{\underline{T}}^{b_j o}_i + \sum_{K=1}^{NM^j} \underline{\underline{T}}^{b_j o}_{Ki} \eta^j_K + \sum_{K=1}^{NM^j} \sum_{M=1}^{NM^j} \underline{\underline{T}}^{b_j o}_{MKi} \eta^j_m \eta^j_K \right) \cdot (\underline{\omega}^j + \underline{\mathring{u}}'^j_o)$$

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$$(\text{VI.})\quad \underline{\omega}^{j} \cdot \left\{ \sum_{k=1}^{NM_j} \underline{W}^{j}_{ki}\, \dot{\eta}^{j}_{k} + \sum_{k=1}^{NM_j} \sum_{m=1}^{NM_j} \underline{W}^{j}_{mki}\, \eta^{j}_{m}\, \dot{\eta}^{j}_{k} \right\}$$

$$(\text{VII.})\quad \sum_{o=1}^{NM_j} \left(\underline{\omega}^{j} \times \mathring{\underline{u}}'^{j}_{o} \right) \cdot \left\{ \underline{D}^{b_j o}_{i} + \sum_{k=1}^{NM_j} \underline{D}^{b_j o}_{ki}\, \eta^{j}_{k} + \sum_{k=1}^{NM_j} \sum_{m=1}^{NM_j} \underline{D}^{b_j o}_{mki}\, \eta^{j}_{m}\, \eta^{j}_{k} \right\}$$

RATE OF CHANGE OF BODY j ANGULAR MOMENTUM
(CONSOLIDATED EXPRESSION FOR $\dot{\underline{H}}^{j}$)

$$\dot{\underline{H}}^{j} = \underline{\underline{I}}^{j\prime} \cdot \dot{\underline{\omega}}^{j} + \sum_{i=1}^{NM_j} \left(\underline{h}^{j}_{i} + \sum_{k=1}^{NM_j} \underline{Y}^{j}_{ki}\, \eta^{j}_{k} + \sum_{k=1}^{NM_j} \sum_{m=1}^{NM_j} \underline{Z}^{j}_{mki}\, \eta^{j}_{m}\, \eta^{j}_{k} \right) \ddot{\eta}^{j}_{i} + \underline{\omega}^{j} \times \underline{\underline{I}}^{j\prime} \cdot \underline{\omega}^{j}$$

$$+ \left[2 \sum_{i=1}^{NM_j} \left\{ \underline{\underline{M}}^{j}_{i} + \sum_{k=1}^{NM_j} \underline{\underline{P}}^{j}_{ki}\, \eta^{j}_{k} \right\} \dot{\eta}^{j}_{i} + \underline{\underline{K}}^{j} \right] \cdot \underline{\omega}^{j} + \underline{R}^{j}$$

WHERE :

$$\underline{\underline{I}}^{j\prime} = \underline{\underline{I}}^{j}_{R} + \sum_{i=1}^{NM_j} \left\{ \underline{\underline{M}}^{j}_{i} + \underline{\underline{N}}^{j}_{i} \right\} \eta^{j}_{i} + \sum_{i=1}^{NM_j} \sum_{k=1}^{NM_j} \underline{\underline{P}}^{j}_{ki}\, \eta^{j}_{k}\, \eta^{j}_{i}$$

DEFINITION OF VECTORS AND DYADICS

DEFINE : $\mathbf{b}^j = \begin{Bmatrix} \underline{b}_{j1} \\ \underline{b}_{j2} \\ \underline{b}_{j3} \end{Bmatrix}$; $\mathbf{b}^{jT} = \{ \underline{b}_{j1} \; \underline{b}_{j2} \; \underline{b}_{j3} \}$, BODY j REFERENCE BASIS

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\alpha}^j_i$	0	MODAL	$\frac{1}{m_j} \left\{ \sum_{o=1}^{NNB_j} m^j_o \underline{\phi}^j_{oi} - m^j_o \underline{l}^j_o \times \underline{\phi}'^j_{oi} - \sum_{k=1}^{NM_j} m^j_o (\underline{\phi}'^j_{ok} \times \underline{l}^j_o) \times \underline{\phi}'^j_{oi} \} \eta^j_k \right\}$
$\underline{\underline{I}}^{b_{jo}}$	0,1,½,2	ROTATIONAL/ MODAL	$\mathbf{b}^T \{ J^{b_{jo}} + \tilde{U}'^j_o J^{b_{jo}} - J^{b_{jo}} \tilde{U}'^j_o - \tilde{U}'^j_o J^{b_{jo}} \tilde{U}'^j_o \} \mathbf{b}$ $\boxed{J^{b_{jo}} \text{ IS INERTIA MATRIX } (3\times3) \text{ OF NODAL BODY } j_o \text{ WRT NODAL BODY } j_o \text{ REFERENCE FRAME }, b_{jo}}$
$\underline{h}^j_i$	0	ROTATIONAL/ MODAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNB_j} \left[m^j_o \tilde{r}^j_o \{\phi^j_{oi}\} - \tilde{r}^j_o m^j_o \tilde{l}^j_o \{\phi'^j_{oi}\} + m^j_o \tilde{l}^j_o \{\phi^j_{oi}\} + J^{b_{jo}} \{\phi'^j_{oi}\} \right] \right\}$ $\boxed{\{x\} \text{ DENOTES COLUMN MATRIX}}$

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{Y}_{ki}^{j}$		ROTATIONAL/ MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[m_o^j\tilde{\phi}_{ok}^{j}\{\phi_{oi}^{j}\} + \tilde{r}_o^j\widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{ok}\})}\{\phi'^{j}_{oi}\} - \tilde{\phi}_{ok}^{j}m_o^j\tilde{l}_o^j\{\phi'^{j}_{oi}\} - m_o^j\widetilde{(\tilde{l}_o^j\{\phi'^{j}_{ok}\})}\{\phi_{oi}^{j}\} + (\tilde{\phi}'^{j}_{ok}J^{b_o o} - J^{b_o o}\tilde{\phi}'^{j}_{ok})\{\phi'^{j}_{oi}\}\right]\right\}$
$\underline{Z}_{mki}^{j}$		ROTATIONAL/ MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[\tilde{\phi}_{om}^{j}\widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{ok}\})}\{\phi'^{j}_{oi}\} - \tilde{\phi}'^{j}_{om}J^{b_o o}\tilde{\phi}'^{j}_{ok}\{\phi'^{j}_{oi}\}\right]\right\}$
$\underline{Y}_{ki}^{j}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[m_o^j\tilde{\phi}_{ok}^{j}\{\phi_{oi}^{j}\} - \tilde{\phi}_{ok}^{j}m_o^j\tilde{l}_o^j\{\phi'^{j}_{oi}\}\right]\right\}$
$\underline{Y}_{mki}^{j}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[\tilde{\phi}_{ok}^{j}\widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{om}\})}\{\phi'^{j}_{oi}\}\right]\right\}$
$\underline{\underline{W}}_{i}^{j}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[m_o^j\tilde{\phi}_{oi}^{j}\tilde{r}_o^j - \widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{oi}\})}\tilde{r}_o^j\right]\right\}\mathbf{b}$
$\underline{\underline{W}}_{ki}^{j}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB^j}\left[\widetilde{\left[\widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{ok}\})}\{\phi'^{j}_{oi}\}\right]}\tilde{r}_o^j + m_o^j\tilde{\phi}_{oi}^{j}\tilde{\phi}_{ok}^{j} - \widetilde{(m_o^j\tilde{l}_o^j\{\phi'^{j}_{oi}\})}\tilde{\phi}_{ok}^{j}\right]\right\}\mathbf{b}$

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VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\underline{W}}^{j}_{mki}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB_j}\left[\left[\left(m_o^j \tilde{l}_o^j \{\phi^{pj}_{ok}\}\right)^{\sim}\{\phi^{pj}_{oi}\}\right]^{\sim}\tilde{\phi}^{j}_{om}\right]\right\}\mathbf{b}$
$\underline{\underline{T}}^{b_jo}_{i}$		MODAL	$\mathbf{b}^T\left\{\tilde{\phi}^{j}_{oi}\, m_o^j \tilde{l}_o^j + J^{b_jo}\,\tilde{\phi}^{pj}_{oi}\right\}\mathbf{b}$
$\underline{\underline{T}}^{b_jo}_{ki}$		MODAL	$\mathbf{b}^T\left\{-\tilde{\phi}^{j}_{oi}\left(m_o^j \tilde{l}_o^j\{\phi^{pj}_{ok}\}\right)^{\sim} + \tilde{\phi}^{pj}_{ok} J^{b_jo}\tilde{\phi}^{pj}_{oi} - J^{b_jo}\tilde{\phi}^{pj}_{ok}\tilde{\phi}^{pj}_{oi}\right\}\mathbf{b}$
$\underline{\underline{T}}^{b_jo}_{mki}$		MODAL	$\mathbf{b}^T\left\{-\tilde{\phi}^{pj}_{om} J^{b_jo}\tilde{\phi}^{pj}_{ok}\tilde{\phi}^{pj}_{oi}\right\}\mathbf{b}$
$\underline{W}^{j}_{ki}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB_j}\left[\tilde{\phi}^{pj}_{ok}\, m_o^j \tilde{l}_o^j\{\phi^{j}_{oi}\}\right]\right\}$
$\underline{W}^{j}_{mki}$		MODAL	$\mathbf{b}^T\left\{\sum_{o=1}^{NNB_j}\left[-\tilde{\phi}^{pj}_{ok}\left(m_o^j \tilde{l}_o^j\{\phi^{pj}_{om}\}\right)^{\sim}\{\phi^{pj}_{oi}\}\right]\right\}$
$\underline{D}^{b_jo}_{i}$		MODAL	$\mathbf{b}^T\left\{J^{b_jo}\{\phi^{pj}_{oi}\}\right\}$
$\underline{D}^{b_jo}_{ki}$		MODAL	$\mathbf{b}^T\left\{\tilde{\phi}^{pj}_{ok} J^{b_jo}\{\phi^{pj}_{oi}\} - J^{b_jo}\tilde{\phi}^{pj}_{ok}\{\phi^{pj}_{oi}\}\right\}$
$\underline{D}^{b_jo}_{mki}$		MODAL	$\mathbf{b}^T\left\{-\tilde{\phi}^{pj}_{om} J^{b_jo}\tilde{\phi}^{pj}_{ok}\{\phi^{pj}_{oi}\}\right\}$

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\underline{I}}^{j}_{R}$	0	ROTATIONAL	$\mathbb{b}^{T}\left\{\sum_{o=1}^{NNB_j}\left[\underline{\underline{J}}^{b_j o}+m_o^j\,\tilde{r}_o^j\,\tilde{r}_o^j-\tilde{r}_o^j\,m_o^j\,\tilde{\ell}_o^j-m_o^j\,\tilde{\ell}_o^j\,\tilde{r}_o^j\right]\right\}\mathbb{b}$
$\underline{\underline{M}}^{j}_{i}$	1	ROTATIONAL	$\mathbb{b}^{T}\left\{\sum_{o=1}^{NNB_j}\left[-m_o^j\,\tilde{r}_o^j\,\tilde{\phi}_{oi}^{j}+\tilde{r}_o^j\,\widetilde{(m_o^j\,\tilde{\ell}_o^j\{\phi_{oi}^{\prime j}\})}-m_o^j\,\tilde{\ell}_o^j\,\tilde{\phi}_{oi}^{j}-\underline{\underline{J}}^{b_j o}\,\tilde{\phi}_{oi}^{\prime j}\right]\right\}\mathbb{b}$
$\underline{\underline{N}}^{j}_{i}$	1	ROTATIONAL	$\mathbb{b}^{T}\left\{\sum_{o=1}^{NNB_j}\left[-m_o^j\,\tilde{\phi}_{oi}^{j}\,\tilde{r}_o^j-\tilde{\phi}_{oi}^{j}\,m_o^j\,\tilde{\ell}_o^j+\widetilde{(m_o^j\,\tilde{\ell}_o^j\{\phi_{oi}^{\prime j}\})}\,\tilde{r}_o^j+\tilde{\phi}_{oi}^{\prime j}\,\underline{\underline{J}}^{b_j o}\right]\right\}\mathbb{b}$
$\underline{\underline{P}}^{j}_{ki}$	2	ROTATIONAL	$\mathbb{b}^{T}\left\{\sum_{o=1}^{NNB_j}\left[-m_o^j\,\tilde{\phi}_{ok}^{j}\,\tilde{\phi}_{oi}^{j}+\tilde{\phi}_{ok}^{j}\,\widetilde{(m_o^j\,\tilde{\ell}_o^j\{\phi_{oi}^{\prime j}\})}+\widetilde{(m_o^j\,\tilde{\ell}_o^j\{\phi_{ok}^{\prime j}\})}\,\tilde{\phi}_{oi}^{j}-\tilde{\phi}_{ok}^{\prime j}\,\underline{\underline{J}}^{b_j o}\,\tilde{\phi}_{oi}^{\prime j}\right]\right\}\mathbb{b}$
$\underline{\underline{K}}^{j}$	MULTIPLE	ROTATIONAL	$\underline{\underline{K}}^{j}=\sum_{i=1}^{NM_j}\underline{\underline{K}}^{j}_{i}\,\dot{\eta}_i^j+\sum_{i=1}^{NM_j}\sum_{k=1}^{NM_j}\underline{\underline{K}}^{j}_{ki}\,\eta_k^j\,\dot{\eta}_i^j+\sum_{i=1}^{NM_j}\sum_{k=1}^{NM_j}\sum_{m=1}^{NM_j}\underline{\underline{K}}^{j}_{mki}\,\eta_m^j\,\eta_k^j\,\dot{\eta}_i^j$

Dynacs

VARIABLE	ORDER	D.O.F. ASSOCIATION	DEFINITION
$\underline{\underline{K}}^{j}_{i}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNB^j} \left[J^{b_i o} \tilde{\phi}^{\prime j}_{oi} + \tilde{\phi}^{\prime j}_{oi} J^{b_i o} \right] \right\} \mathbf{b}$
$\underline{\underline{K}}^{j}_{ki}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNB^j} \left[\tilde{\phi}^{\prime j}_{ok} J^{b_i o} \tilde{\phi}^{\prime j}_{oi} + J^{b_i o} \tilde{\phi}^{\prime j}_{ok} \tilde{\phi}^{\prime j}_{oi} + \tilde{\phi}^{\prime j}_{oi} \tilde{\phi}^{\prime j}_{ok} J^{b_i o} - \tilde{\phi}^{\prime j}_{oi} J^{b_i o} \tilde{\phi}^{\prime j}_{ok} + 2 \tilde{r}^{j}_{o} \left[\widetilde{\tilde{\phi}^{\prime j}_{oi} (m^j_o \tilde{l}^j_o \{\phi^{\prime j}_{ok}\})} \right] \right\} \mathbf{b}$
$\underline{\underline{K}}^{j}_{mki}$		ROTATIONAL	$\mathbf{b}^T \left\{ \sum_{o=1}^{NNB^j} \left[2 \tilde{\phi}^{\prime j}_{ok} \left[\widetilde{\tilde{\phi}^{\prime j}_{oi} (m^j_o \tilde{l}^j_o \{\phi^{\prime j}_{om}\})} \right] + \tilde{\phi}^{\prime j}_{om} J^{b_i o} \tilde{\phi}^{\prime j}_{ok} \tilde{\phi}^{\prime j}_{oi} - \tilde{\phi}^{\prime j}_{oi} \tilde{\phi}^{\prime j}_{ok} J^{b_i o} \tilde{\phi}^{\prime j}_{om} \right] \right\} \mathbf{b}$
$\underline{R}^{j}$	MULTIPLE	ROTATIONAL	$\sum_{o=1}^{NNB^j} \left\{ \underline{\omega}^j \times \underline{\underline{I}}^{b_i o} \cdot \mathring{\underline{u}}^{\prime j}_{o} + \mathring{\underline{u}}^{\prime j}_{o} \times \underline{\underline{I}}^{b_i o} \cdot \mathring{\underline{u}}^{\prime j}_{o} + (\underline{r}^j_o + \underline{u}^j_o) \times (\mathring{\underline{u}}^{\prime j}_{o} \times (\mathring{\underline{u}}^{\prime j}_{o} \times (m^j_o \underline{l}^j_o + \underline{u}^{\prime j}_{o} \times m^j_o \underline{l}^j_o))) \right\}$